Cosmological N-body simulations and SDSS galaxy catalog are used to study the structure of the dark matter halos, the distribution of sub halos inside main halos and the correspondence between dark matter halos and galaxies. The main assumption in this study is that each sub halo contains one galaxy and the simple analytical equation can be used to assign luminosities for dark matter halos. By comparing the properties of galaxy groups in SDSS and simulations we test the hypothesis that galaxies are formed within large halos in a similar manner as sub halos are formed inside main halos. Four different volume limited samples are derived from the galaxy group catalog SDSS DR6. Specifically, we compare the group richness, virial radius, maximum separation and velocity dispersion distributions and find a good agreement between the mock catalogs and observations. This work strongly supports the idea that the link between galaxies in galaxy groups and dark matter sub halos inside main halos is very strong.

1. Introduction

If the concordance LCDM model is a true description of the universe, it should also properly predict the properties and structure of dark matter halos, where galaxies are born. The best-known observational counterparts of massive dark halos are clusters of galaxies. These are the most massive and the largest gravitationally bound systems known to exist in the Universe. Being the vanguard in non-linear regime, clusters of galaxies are important link between the initial density field and present day structures in the Universe. Recent numerical and analytical studies of the cluster scale dark matter (DM) halos agree well with observed cluster abundances [1, 2, 3]. Also many studies about the large scale galaxy clustering give similar results. Two-point correlation functions calculated from the halos in ΛCDM-simulations and galaxies from SDSS agree very well [4] and similar results are obtained from Virgo Consortium simulations in larger scales [5]. Also the galaxy formation physics incorporated in the SPH simulation give a good account of observed galaxy clustering [6].

As a subsequent step, high-resolution numerical studies are pushing the theory of the structure formation to smaller scales toward the 'galactic' sub halo region (sub halos are halos within the virial radius of the main halo) [7, 8, 9, 10]. This is an important step because the substructure of large DM halos links cluster halos and galaxy halos together, providing observable probes for structure formation scenarios. Probably the most difficult problem at small scales is the so-called 'dwarf galaxy crisis'; simulations predict substantially more substructure within the galactic DM halos than observed [11]. Basically all cosmological simulations predict that there are at least one order of magnitude more small sub halos (dwarf galaxies) around Milky Way like galaxies than what is observed (e.g. Via Lactea simulation [12]).

Although the idea of galaxy-dark matter halo connection is well justified, the question how tight the connection is and what properties are related, remains open. This question is closely related to the question how galaxies and galaxy groups are formed and structured. By using N-body simulations with a broad scale of mass and spatial resolution, we study the structure of dark matter halos and the properties of galaxy groups assuming that the galaxy group is defined as a main halo – sub halo system. We test the hypothesis that galaxies are formed within the large halos as sub halos to the main halo, by comparing the simulations against galaxy group data obtained from the Sloan Digital Sky Survey sixth data release (SDSS DR6) [http://www.sdss.org/dr6/] that contains 790860 galaxies and covers ~17% of the full sky. Four different volume limited samples are derived from the data by using a constant linking length within the sample, but different in samples. A similar galaxy group information is calculated as in Tago et al. (2008) galaxy group catalog [13]. Specifically, we compare the group richness, velocity dispersion, virial radius and maximum projected separation distributions of the groups and study the differences between the mock catalogs and observations. We always consider a galaxy group as a system with the main halo and its sub halos.
The closest study similar to us is Berlind et al. (2006), where they used N-body simulations to find the most appropriate linking lengths (projected and line-of-sight) that would find galaxy groups in SDSS data in the best way [14]. They also used different recipes to populate halos with galaxies. From this study the most interesting result for us is the multiplicity function that we can compare to our results. The fundamental difference between this work and [14] is that here we analyze several volume limited samples and study also other properties of galaxy groups. Our method of defining the galaxy group in simulations is completely different and is not based on the friends-of-friends (FoF) method that has been widely used also in simulations.

2. Mock catalogs

We carried out three $\Lambda$CDM simulations with different resolutions using the GADGET-2 code [15]. We have chosen the chain of resolutions to be able to systematically evaluate halo properties in different simulations. By combining these results, we can estimate and eliminate the resolution effects and we can study different absolute magnitude intervals for galaxy comparison. Dark matter halos are identified using an algorithm, called AHF, that is based on the adaptive grid structure of the simulation code in AMIGA [16]. To be relatively conservative in our calculations, individual halos have to contain at least $10^2$ particles to be registered as a real halo. For the Gadget runs we have adopted the currently popular cosmological model $\Lambda$CDM with $(\Omega_{dm} + \Omega_b + \Omega_{\Lambda})=1$ and $h=0.71$ (here and throughout this paper $h$ is the present-day Hubble constant in units of $100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$). All simulations have 512$^3$ dark matter particles and baryonic content is ignored. The simulation volumes employed are $(20\,h^{-1}\,\text{Mpc})^3$, $(40\,h^{-1}\,\text{Mpc})^3$ and $(80\,h^{-1}\,\text{Mpc})^3$. We call these simulations as B20, B40 and B80. Mass resolutions are $4.47 \times 10^6\,h^{-1}\,\text{M}_\odot$, $3.57 \times 10^7\,h^{-1}\,\text{M}_\odot$ and $2.86 \times 10^8\,h^{-1}\,\text{M}_\odot$, respectively. The adopted cosmological parameters agree with the results by WMAP microwave background anisotropy experiment team [17]. The transfer function and the initial data for our models were computed using the COSMIX code by E.Bertschinger (http://arcturus.mit.edu/cosmics/).

There is no unique way to calculate luminosities of galaxies starting from dark matter halos. We apply a simplified method to assign a luminosity to dark matter halo for which virial mass is known. The method is based on the analytical equation that was derived by Vale & Ostriker [18]. This function gives a dark matter halo – luminosity relation $L_b(M_{dm})$, but ignores the realistic scatter in luminosity at given mass. Since our procedure averages all different halos, the scatter is also averaged out, but this is not important in our case.

To test our results against other mock catalogues we include data from the Millennium Simulation [19, hereafter MS]. MS is a cosmological N-body simulation of the $\Lambda$CDM model performed by Virgo Consortium and it was carried out with a customized version of the GADGET2 code. The MS follows the evolution of 2150$^3$ particles from redshift $z = 127$ in a box of 500$h^{-1}\,\text{Mpc}$ on a side. The cosmological parameters of the MS simulation are: $\Omega_m=\Omega_{dm}+\Omega_b=0.25$, $\Omega_b=0.045$, $h=0.73$, $\Omega_{\Lambda}=0.75$, $n=1$, and $\sigma_8=0.9$. The mass resolution is about three times worse than in our B80 simulation. The simulated galaxies in the MS data are based on merger trees and their properties are obtained by using semi-analytic galaxy formation models, where the star formation and its regulation by feedback processes are parameterized in terms of analytical physical models. This is the main difference between our simulations and MS, and very likely is the main factor that causes different results in comparison data. More information about the MS group catalog is given in [20, 21].

3. Comparison with observations

To study the properties of galaxy groups we have selected different volume limited samples from the DR6 data. By using volume selected samples the comparison with the simulations is free of many selection effects and the calculated distributions can be directly compared. Four different absolute magnitude intervals are used: [-18 - -22], [-19 - -22], [-20 - -22] and [-21 - -22], that divide the data to four samples S1, S2, S3 and S4. The observed magnitudes of individual galaxies in R-band $m_r$ are between 12.5 and 17.77. These choices of completeness levels lead to redshift intervals [0.026-0.045], [0.026-0.069], [0.026-0.10] and [0.026-0.15]. These limits correspond the lower limit in distance: 76 Mpc/h and upper limits of 133 Mpc/h, 204 Mpc/h, 305 Mpc/h and 445 Mpc/h, in comoving co-ordinates. After calculating comoving volumes we normalize
the distributions and calculate the space densities of groups, hence direct comparison to simulated data is possible. The number of groups in the simulations varies significantly from 3 to 874 depending on the resolution of the simulations, whereas in the SDSS data numbers are much higher. Space densities of groups and numbers of groups are given in the Table 1. In the highest resolution simulation B20 the number of groups, even at the most extensive sample S1, is too small to include enough data to distributions and internal errors are very large. In all cases the error bars shown are one sigma Poisson errors in the bin. In the logarithmic scale, if there is only one data point in the bin, the minimum of the error bar is zero. In general, B40 and B80 provide enough galaxies to different groups and these can be used for reliable comparison. Their differences give some idea about how large the cosmic variation is.

<table>
<thead>
<tr>
<th>Absolute magnitude interval/Box size</th>
<th>20 Mpc/h</th>
<th>40 Mpc/h</th>
<th>80 Mpc/h</th>
<th>SDSS DR6</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: -18 - -22 Number of groups</td>
<td>22</td>
<td>141</td>
<td>874</td>
<td>4431</td>
</tr>
<tr>
<td>Space density (Mpc/h)^-3</td>
<td>2.75 x10^-3</td>
<td>2.20 x 10^-3</td>
<td>1.71 x 10^-3</td>
<td>3.33 x 10^-3</td>
</tr>
<tr>
<td>S2: -19 - -22 Number of groups</td>
<td>15</td>
<td>85</td>
<td>701</td>
<td>9996</td>
</tr>
<tr>
<td>Space density (Mpc/h)^-3</td>
<td>1.88 x 10^-3</td>
<td>1.33 x 10^-3</td>
<td>1.37 x 10^-3</td>
<td>1.82 x 10^-3</td>
</tr>
<tr>
<td>S3: -20 - -22 Number of groups</td>
<td>10</td>
<td>40</td>
<td>413</td>
<td>15402</td>
</tr>
<tr>
<td>Space density (Mpc/h)^-3</td>
<td>1.25 x 10^-3</td>
<td>6.25 x 10^-4</td>
<td>8.07 x 10^-4</td>
<td>7.92 x 10^-4</td>
</tr>
<tr>
<td>S4: -21 - -22 Number of groups</td>
<td>3</td>
<td>13</td>
<td>132</td>
<td>9581</td>
</tr>
<tr>
<td>Space density (Mpc/h)^-3</td>
<td>3.75 x 10^-4</td>
<td>2.03 x 10^-4</td>
<td>2.58 x 10^-4</td>
<td>1.55 x 10^-4</td>
</tr>
</tbody>
</table>

Table 1 Total number of groups and space densities in SDSS and in our three simulations (Gadget).

Frequency distribution of the group richness (number of members in galaxy groups), is shown in (Fig 1). The symbols for B20 ( hyphen), B40 (cross), B80 (dot) and MS (square) are the same in all figures as is shown in the fourth panel. Multiplicity function measurements provide one of the key constraints on the relation between galaxy populations and dark matter halos. The first bin is for pairs and the distribution extends up to 100 members. There is a clear exponential trend and we fit a straight line to distributions to see how the slope value depends on the absolute magnitude. Berlind et al. (2006) found that multiplicity functions are well fitted by power-law relations, with best-fit slopes of -2.72±0.16, -2.48±0.14, and -2.49±0.28 for absolute magnitude intervals of Mr< -20, -19 and -18 [14]. These absolute magnitude limits are approximately the same as our first three samples. If we fit straight lines to our histogram data, the measured slope values are -2.0, -2.2, -2.6 and -3.4 for Mr< -18, -19, -20 and -21, respectively. Our values are slightly smaller than in [14], but there is a similar trend that for brighter groups the slope is steeper.
Next we have calculated the virial radius distributions (Fig 2). Virial radius is defined as the projected harmonic mean of the separations as measured from the group center. First three samples provide a good agreement between the simulations and SDSS data although in observations some groups with $R_{\text{vir}} > 0.3$ Mpc/h are missing. In the simulations there is not enough data for statistical study (due to small volumes in simulations) at very small separations $R_{\text{vir}} < 10$ kpc, hence those data points are missing. Millenium data still gives the correct order of magnitude of groups even for small galaxy groups. In S4, the simulations give more larger groups, but also the Poissonian errors are large. As one can expect the maximum value in distributions shifts towards larger distances from 0.22 Mpc/h in S1 to 0.56 Mpc/h in S4 indicating that bright groups are statistically larger in size than faint groups.
Another distribution that can be used to study the group sizes is the projected maximum separation, that is the largest projected galaxy pair separation in the group (Fig 3). Distributions do not agree with simulations as well as for $R_{\text{vir}}$. There are very large variations between different resolution simulations and the largest differences come again at large distances. Simulations produce more larger groups. Especially, in Millenium there seems to be large differences and there are much more larger groups in Millenium galaxy data than in SDSS. Maximum values are the same as for virial radius, but in all samples the shape of the distribution is wider and there are groups that extend up to $R_{\text{max}} = 3\, \text{Mpc/h}$. The minimum size of the groups is ~100 kpc/h. In B20 simulation the error bars are very large and the data is unusable for the brightest galaxies.

Fig. 3 A comparison of maximum separation distributions in SDSS and in our three simulations (Gadget) and Millenium.

To compare dynamical properties of the groups, we have calculated the velocity dispersion distributions of the group members (Fig 4). For S1 there is clear abundance of groups around the maximum at 100 km/s. The difference between the simulations and observations get smaller as the absolute magnitude limit is larger and the number of faint galaxies gets smaller. In the last two samples S3 and S4 the agreement is fairly good. Maximum separation distribution shows that groups in S1 are more compact than those in simulations and these groups may be responsible for the differences in velocity dispersion distributions. It is also notable that all simulations agree very well in the first two samples, but in the last two samples the number of groups in Millenium is smaller, but the shape of the distribution is still the same.
Since the correspondence between the simulations and the observed data is so good, it is very likely that the groups in SDSS correspond the main halo-sub halo systems in the simulations. The method used in the observations is a modification of the FoF-method and it is not obvious that the groups that are found correspond the halo galaxy groups in our simulations.

From the simulations we calculated how bound the groups are. The procedure is the same as in [22] and we calculated the total kinetic energy $T$ and total potential energy $V$ of the group. If $T > U$, we define that the system is unbound and otherwise the group is bound. In the Figure 5 we show the ratio $T/U$ in B40 and B80 simulations. The most obvious trend is that the number of unbound groups drops gradually as there are more members in the system. Also, if only bright members are included (last panel), there are very few unbound systems. These groups form a bright and gravitationally well developed system. One can also see that many pairs are unbound, hence they are merely passing by each other.

The fraction of bound systems in B40 is 0.89 and in B80 this is 0.91 for S1. This is the minimum value for both cases and the fraction is almost 100% in S3 and S4, especially if pairs are ignored. This is what we can expect from the simulations, since the groups are derived from the main halo-sub halo systems that are systems where the sub halos reside in much larger main halo which normally is gravitationally bound. These results differ from the values in [22], where simulations were compared to galaxy catalogs of nearby galaxies: Huchra & Geller (1982) catalog and its extended version UZC-SSRS2 catalog [23]. The main difference between this study and [22] is that here the studied volume is much larger and the groups that are studied are more compact, more bright and does not contain that many members. Also the FoF method used for galaxies in SDSS is completely different and here we select volume limited samples instead of magnitude limited groups in [22]. These distinctions can explain why our halo-sub halo data agrees so well with the SDSS and also these groups may be gravitationally bound “real” groups.
Our method to assign a luminosity to a halo with given mass is very straightforward and therefore we show the distributions of the total luminosities of the group (sum of all members) and compare these with simulations (Fig 6). The lower limit of the group luminosity is determined by the luminosity limit of galaxies and therefore the distribution gets more narrow for bright groups. In observations there are systematically more faint groups and less bright groups than in our simulations. For bright groups Millennium data gives better agreement. Both simulations produce too few faint groups indicating that the halo-luminosity correspondence could be improved. The fact that our simulations give too bright groups may be due to the wrong luminosity of the central galaxy that is the same as the main halo luminosity, in our simulations. This difference can be significant for massive groups where the luminosity of the main galaxy should be smaller than the one calculated based on the main halo mass, since the main halo mass contains all dark matter mass that is not included in the sub halos.

Fig. 5 Boundness (T/U) as a function of richness in B80 and B40 simulations.

Fig. 6 R-band luminosity function of the groups in SDSS data and simulations. See designations in Fig 1.
Conclusions and discussion

The volume limited samples in SDSS allows us to calculate the space densities of the groups and compare the groups properties directly with the simulations. We use the results from three cosmological $\Lambda$CDM N-body simulations and Millenium simulation and conclude that both methods can produce similar distributions although some differences are seen. The richness distribution agrees very well with the simulations and SDSS. Even the number of pairs match well. This supports very strongly the idea that in galaxy groups galaxies and dark matter sub halos are derived from the same distribution. Also the virial radius distribution is very similar confirming the idea that they represent similar groups. Some discrepancies are seen in the maximum separation and velocity dispersion distributions. Differences are larger when faint galaxies are included. This could be explained if we assume that the galaxy distribution inside the galaxy group is different than the sub halo distribution in the main halo. The differences in the luminosity distribution indicate that the connection between the halo mass and galaxy luminosity may not be that straightforward as we have assumed. One possible explanation for the observed differences in $R_{\text{vir}}$ and $R_{\text{max}}$ may be related to our simple approach to populate dark matter halos with galaxies. It is possible that there are systematic differences in the luminosities of galaxies based on the position in the group. If galaxies of the same mass would be systematically brighter close to the core of the group, then the observed large and bright groups would be more compact than in simulations with random sampling. This could also affect the velocity dispersion distribution and lead to smaller velocity dispersions than in simulated data. In our simulation the groups are mostly gravitationally bound and the same may hold for the observed groups.

Despite of the found differences, such a good comparison result gives a good support for the idea that galaxies are formed within the massive halo and later form a bound galaxy groups. The system evolves in time and these evolutionary effects may be important for the galaxy luminosity. This study gives a good background for more detailed studies about the evolution of groups in simulations and encourages to study galaxy groups directly as main halo-sub halo systems.

References